THE BRITISH SOCIETY FOR THE HISTORY OF MATHEMATICS
Bulletin 4 Spring 2005

Influence, ignorance, or indifference? Rethinking the relationship between Babylonian and Greek mathematics

Eleanor Robson[1]

This talk is about beginnings and endings. It's about the origins of Greek mathematics, and the end — or even *ends* — of it in Babylonia. It's about the origins of David's calling as a historian, and of mine. But it is also about the end of David's career and life, in particular what we were working on together at the time.

David was very partial to origins stories — indeed, as we shall see, his career as a historian was largely built around them — so I will start by telling you mine. I was a maths undergraduate at Warwick in the late 80s and chose in my final year, for complicated reasons that are now rather hard to construct, to take the history of maths course that David ran (see Fowler 1992; Lehmann 1995). What I do remember, most vividly, was getting hooked on the early Greek maths that David assigned as holiday reading, and then volunteering to give one of the first compulsory ten-minute presentations in the lectures themselves — not least because I had a dread of speaking in public and wanted to get it out of the way as soon as possible. I spent a week in the library researching my topic and then gave a talk that lasted nearer thirty minutes than ten. I had fallen in love, irrevocably, with Babylonian mathematics. David took my obsession seriously and encouraged me to learn the script, languages, history, and archaeology necessary to study my beloved subject. We hooked up again in 1994, at a BSHM meeting on non-European maths, as I was finishing my doctorate in Oriental Studies at Oxford. And then he really started to teach me, and has continued to do so ever since. Not just about ancient mathematics, or about writing and public speaking, but about life, death, and friendship.

Now older and wiser than I was on the occasion of my first ever talk for David, I promise not to overrun my time today. I want to use the next 45 minutes to describe some elements of David's contribution to the history of ancient mathematics, and to share with you some ideas we were working on together in the last few years.

Influence: an influential theory and a controversial refutation

Many of you know the standard story of early Greek mathematics, and David's radical revision of it, as told in his magisterial book, *The mathematics of Plato's academy* (Fowler 1987/1999). Nevertheless it may be useful to recap it now, highlighting especially what the traditional story and David's each have to say about the relationship

1 Department of History and Philosophy of Science, University of Cambridge, Free School Lane, Cambridge CB2 3RH, UK <er264@cam.ac.uk>

Front cover illustration: Germinal Pierre Dandelin's (1794–1847) long-neglected theorem on spheres, cones, and conic sections is revived by Elizabeth Boag on p 22.

between mathematics in Greece and Babylonia. I am going to do this in the manner of many Late Antique commentators on Greek mathematics: first I shall quote at length from a work by my teacher, and then I shall add scholia and glosses to elaborate and expand on his original text. I have chosen six key paragraphs from one of the last articles David published, which came out in *Revue d'Histoire des Mathématiques* in 1999. It is called 'Inventive Interpretations' and is a succinct summary of his thesis. These are the opening sentences:

> Suppose we are investigating some substantial corpus of historical texts in their final published version, without any access to notes, drafts, letters, conversations, etc. regarding the composition of this material. Also suppose that, in this corpus, we find no reference whatsoever to some feature that we regard as intimately connected with the subject matter, something indeed that we find hard to put aside, or that we may not even realise is only in our mind. How might we react to this? (Fowler 1999: 149)

This is David in typically Socratic mode: he poses a hypothetical, provocative question, asks the reader to carry out a thought experiment. Notice that David makes no mention of mathematics or Greek: all we know about this historical corpus is that it counter-intuitively lacks some key characteristic that we had assumed it would feature. David invites us, very gently, to reconsider our assumptions.

Now, this seems a reasonable course of action, so David now reveals the identity of the corpus, and its missing component:

> The paradigm example I am thinking of here is Greek mathematics, in particular the deductive geometry of Euclid, Archimedes, and Apollonius and, to a lesser extent, parts of some later mathematicians and commentators. […] And the missing topic in this synthetic geometry is any involvement with any kind of numbers beyond the *arithmoi*, 1, 2, 3, … , conceived very concretely. (Fowler 1999: 149)

To illustrate this, let's take an example of deductive Greek mathematics, chosen by David himself (Example 1a). Proposition 5 of Book II of Euclid's *Elements* was written in Alexandria in about 300 BCE. Without reading it in detail, we notice that this is a geometry without numbers: the points, lines, and areas are described by letters but have no particular sizes attached to them. David contrasts this style with the modern impulse to describe mathematical objects arithmetically and algebraically —as, for instance $lw + 1/4(l - w)^2 = 1/4(l + w)^2$ for *EE* II.5 — and thus the tendency of nineteenth and twentieth-century translators and commentators to represent Greek mathematics in symbolic algebra. Then he asks:

> What, then, is the principal interpretation today of this feature, the complete absence of anything like the rational or real numbers from this corpus? To describe it briefly and brutally, it is that the early Greek mathematicians did start with an arithmetised geometry, but then reformulated it in this non-arithmetical way. Moreover, though this is not often said, they did this so thoroughly that no trace of this earlier arithmetic survives in this corpus of deductive mathematics. In other words, we have here an invention of its supposed presence, followed by an invention of its complete disappearance! (Fowler 1999: 150–1)

In the standard account, this reformulation grew out of the so-called crisis of incommensurability, when Pythagoras and his followers were grappling with the realisation that not all lines could be measured in terms of any other line. The diagonal of a square, for instance, was not arithmetically expressible in terms of the length of that square. Thus the Pythagoreans gave up in disgust and retreated to the safety of exploring the properties and relationships of points, lines, and areas without recourse to badly behaved numbers. As David acknowledged:

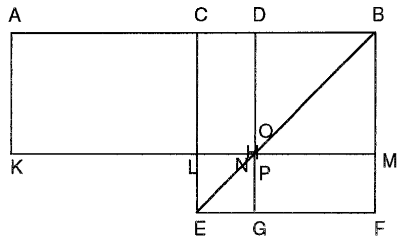
> Of course there are reasons for these proposals. First there is the existence of the earlier sophisticated Babylonian mathematics, which is thoroughly and visibly arithmetised, though more subtly and less obviously geometrical. Here the next invention of modern scholarship is of a Babylonian influence on the origins of Greek deductive mathematics, and hence its arithmetical basis at this earliest stage, and again I say 'invention' because the evidence is indirect, mainly by comparing some Babylonian procedures with those of some Euclidean propositions. But, just to point to one missing feature, there is no evidence known so far of any sexagesimal numbers, the formulation of Babylonian mathematics, to be found in Greek scientific thought before the time of Hypsicles in the second century BC. (Fowler 1999: 151)

Example 1a: Elements *II.5, Alexandria, Egypt, c.300 BCE (trans. Heath 1926)*

I have simplified some of Heath's formatting.

If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half.

For let a straight line *AB* be cut into equal segments at *C* and into unequal segments at *D*; I say that the rectangle contained by *AD*, *DB* together with the square on *CD* is equal to the square on *CB*.



For let the square *CEFB* be described on *CB*, [I. 46] and let *BE* be joined; through *D* let *DG* be drawn parallel to either *CE* or *BF*, through *H* again let *KM* be drawn parallel to either *AB* or *EF*, and again through *A* let *AK* be drawn parallel to either *CL* or *BM*. [I. 31]

Then, since the complement *CH* is equal to the complement *HF*, [I. 43] let *DM* be added to each; therefore the whole *CM* is equal to the whole *DF*.

But *CM* is equal to *AL*, since *AC* is also equal to *CB*; [I. 36] therefore *AL* is also equal to *DF*.

Let *CH* be added to each; therefore the whole *AH* is equal to the gnomon *NOP*.

But *AH* is the rectangle *AD*, *DB*, for *DH* is equal to *DB*, therefore the gnomon *NOP* is also equal to the rectangle *AD*, *DB*.

Let *LG*, which is equal to the square on *CD*, be added to each; therefore the gnomon *NOP* and *LG* are equal to the rectangle contained by *AD*, *DB* and the square on *CD*.

But the gnomon *NOP* and *LG* are the whole square *CEFB*, which is described on *CB*; therefore the rectangle contained by *AD*, *DB* together with the square on *CD* is equal to the square on *CB*.

Therefore etc. Q. E. D.

***Example 2a:** AO 8862, Larsa, southern Iraq, c.1750 BCE (trans. and comm. Van der Waerden 1954: 63)*[2]

(1) Length, width. I have multiplied length and width, thus obtaining the area. Then I added to the area, the excess of the length over the width: 3 03 (i.e., 183 was the result). Moreover, I have added the length and width: 27. Required length, width, and area.
(given:) 27 and 3 02, the sums
(result:) 15 length 3 00, area
12 width
One follows this method:

27 + 3 03 = 3 30
2 + 27 = 29

Take one half of 29 (this gives 14;30).

14;30 x 14;30 = 3 30;15
3 30;15 – 3 30 = 0;15

The square root of 0;15 is 0;30.

14;30 + 0;30 = 15 length
14;30 – 0;30 = 14 width.

Subtract 2, which has been added to 27, from 14, the width. 12 is the actual width.
I have multiplied 15 length by 12 width.

15 x 12 = 3 00 area
15 – 12 = 3
3 00 + 3 = 3 03.

The first lines formulate the problem: two equations with two unknowns, each represented by a symbol, ush and sag, length and width. The Sumerian symbols are dealt with as our algebraic symbols x and y; they possess the same advantages of remaining unchanged in declension. We can therefore safely put the problem in the form of 2 algebraic equations:

(1) $xy + x - y = 183$
$x + y = 2$

The last 4 lines of the text merely verify that the resulting numbers $x = 15$ and $y = 12$ indeed satisfy (1). […] What the Babylonians do, step by step, in numbers, amounts indeed to application of the formulas. […] But they do not give these formulas; they merely give one example after another, each of which illustrates the same method of calculation.

As Babylonian mathematics began to be discovered and interpreted in the 1920s and 30s it was translated and analysed in the same modernising style as Greek maths had been a generation earlier. The first four propositions of Book II of Euclid's *Elements* were found to have Babylonian precursors, as did 'Pythagoras' theorem'. Most recently Høyrup (2002: 400–405) argues that Euclid's *Elements* II.1–10, VI.28–29, *Data* 84–86, and Diophantos' *Arithmetic* I.27–30 were all influenced by Old Babylonian mathematics. Let's give another example: a problem that David picked out because, like

2 Here the sexagesimal place value system is transcribed with spaces separating sexagesimal places and semicolons marking the boundary between integers and fractions.

Elements II.5, it deals with the difference between the length and width of a rectangle (Example 2a). Here the question is formulated explicitly with numbers, and numbers are used throughout the solution. Conversely, there is no overt reference to diagrams, points, or lines.

> Now [says David,] the disappearance. Here, again briefly and brutally, the explanation is that the discovery that √2 is an irrational number put in question this arithmetised approach to geometry, so the subject had to be reformulated non-arithmetically. There is evidence for this story, but all of it is late (fourth century AD onwards) and found only in generally unreliable sources (mainly Iamblicus). (Fowler 1999: 152)

Here is a key passage from Iamblichus's *On the Pythagorean life*, written, remember, nearly a millennium after the time of Pythagoras (see Cuomo 2001: 50–61):

> He [Pythagoras] was captured by the expedition of Kambyses and taken to Babylon. There he spent time with the Magi, to their mutual rejoicing, learning what was holy among them, acquiring perfected knowledge of the worship of the gods and reaching the heights of their mathematics and music and other disciplines. (Iamblichus/Clark 1989: 8 §19)

Over forty years ago Walter Burkert's seminal study, *Lore and science in ancient Pythagoreanism* (Burkert 1962/1972) found that we can confidently attribute to the early Pythagoreans some impressive developments in musical theory and acoustics, but nothing at all in mathematics. More recently D J O'Meara's study of the Neo-Pythagoreans of Late Antiquity has found no trace of concern with commensurability and incommensurability in their writings (O'Meara 1989). David concludes:

> So, in summary, we have very little if any evidence for the influence of Babylonian arithmetical procedures on early Greek mathematics, and only very unreliable evidence concerning its subsequent disappearance. […] Although this interpretation may eventually be found to be justified, at present it is invention, pure and simple. (Fowler 1999: 152)

David's book tackles these huge questions from the standpoint of Greek mathematics, and does so in impressive detail and from many angles. What we were doing more recently, and what I want to explain to you now, is to further debunk the supposed connection between Greek and Babylonian maths, from both sides of the fence. Let me try and summarise that work now. In doing so I shall bring together two of the three important and influential threads in David's thinking: that for historians mathematics should not just be seen as mathematical, but as textual, material, and social too. In short, we are engaged with what people have done with, and thought about, mathematics as much as or even more than the mathematics itself.

Interpretations: textuality and translation

The majority of our evidence for Babylonian mathematics in fact comes from two widely separated time-spans. The first and best known is from the early second millennium BCE, or Old Babylonian period. This is what most people mean when they talk about Babylonian or Mesopotamian mathematics, and it was this mathematics that twentieth-century historians took to have been proto-Greek. Then there is a corpus of much later evidence, from around 650 BCE to perhaps as late as the first century CE. This Later Babylonian mathematics (as I shall call it here) has been largely ignored by twentieth-century historians, though I have made it a particular focus of my research in the last two or three years. We find, as in Example 3a, that many of the problems of Old

Babylonian mathematics remain, but solved in very different ways. Why it has been neglected is a point I shall return to later, because we shall have to take into account a third period of history—the mid-twentieth century—in order to understand how and why Iamblichus' fairy stories were swallowed hook, line, and sinker.

Example 3a: *BM 34568, Babylon, southern Iraq, c.300 BCE (trans. and comm. Neugebauer 1935–37: III, 18–21)*

Translated from the German.

(9). Length and breadth added is 14 and 48 (is) the area. The sizes are not known. 14 times 14 (is) 3 16. 48 times 4 (is) 3 12. You subtract 3 12 from 3 16 and it leaves 4. What times what should I take in order (to get) 4? 2 times 2 (is) 4. You subtract 2 from 14 and it leaves 12. 12 times 0;30 (is) 6. 6 (is) the width. To 2 you shall add 6, it is 8. 8 (is) the length.

(15) The length goes beyond the width by 7 cubits. 2 00 (is) the area. What are length and width? 2 00 times 4 (is) 8 00. 7 times 7 (is) 49. 8 00 and 49 added together (is) 8 49. What times what should I take in order (to get) 8 49? 23 times 23 (is) 8 49. You subtract 7 from 23 and it leaves 16. Take 16 times 0;30. (It is) 8. 8 (is) the width. 7 and 8 added is 15. 15 (is) the length.

(9) From $l + w$ and $A = lw$, w and l are accordingly calculated:

$$w = \tfrac{1}{2}\left((l + w) - \sqrt{(l + w)^2 - 4A}\right)$$

$$l = w + \sqrt{(l + w)^2 - 4A}$$

(15) From $l - w$ and A

$$w = \tfrac{1}{2}\left(\sqrt{4A + (l - w)^2} - (l - w)\right)$$

$$l = (l - b) + b$$

are calculated, which is correct, as $4A - (l - w)^2 = (l + w)^2$.

In a nutshell, I want to argue, with David, that early Greek mathematics was entirely ignorant of its Old Babylonian predecessor and, solely on my own account, that it was indifferent to its Later Babylonian contemporary. To do that I will make my case on two fronts: first through a comparison of the internal features of the three mathematical cultures, and second through an analysis of the social conditions under which the those cultures came into being and died out.

Let's take the internal characteristics first, again based on the three examples I have already used. But in each case I shall give a second translation: the first made in the mid-twentieth century, and the second in the late 1990s (Examples 1b, 2b, 3b). For most of the twentieth century the primary aim of historical research on ancient mathematics was to discover *what* the ancients knew and to analyse it as mathematics pure and simple. As Karin Tybjerg has recently put it:

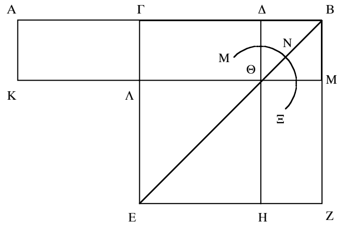
> Treatments such as those of Thomas Heath, Otto Neugebauer and Hieronymus Zeuthen […] were primarily concerned with mathematical content and because content was thought to be independent of presentation, the Greek geometrical texts were not just translated to a modern language, but also rewritten in the modern mathematical idiom of geometrical algebra. (Tybjerg 2005)

Example 1b: Elements *II.5, Alexandria, Egypt, c.300* BCE *(trans. Netz 1999: 1–2)*

Words in angle brackets < > have been added by Netz for ease of comprehension; they are not in the Greek.

If a straight line is cut into equal and unequal <segments>, the rectangle contained by the unequal segments of the whole, with the square on the <line> between the cuts, is equal to the square on the half.

For let some line, <namely> the <line> ΑΒ, be cut into equal <segments> at the <point> Γ, and into unequal <segments> at the <point> Δ;



I say that the rectangle contained by the <lines> ΑΔ, ΔΒ, with the square on the <line> ΓΔ, is equal to the square on the <line> ΓΒ.

For, on the <line> ΓΒ, let a square be set up, <namely> the <square> ΓΕΖΒ, and let the <line> ΒΕ be joined, and, through the <point> Δ, let the <line> ΔΗ be drawn parallel to either of the <lines> ΓΕ, ΒΖ, and, through the <point> Θ, again let the <line> ΚΜ be drawn parallel to either of the <lines> ΑΒ, ΕΖ, and again, through the <point> Α, let the <line> ΑΚ be drawn parallel to either of the <lines> ΓΛ, ΒΜ.

And since the complement ΓΘ is equal to the complement ΘΖ, let the <square> ΔΜ be added <as> common; therefore the whole ΓΜ is equal to the whole ΔΖ. But the <area> ΓΜ is equal to the <area> ΑΛ, since the <line> ΑΓ, too, is equal to the <line> ΓΒ; therefore the <area> ΑΛ, too, is equal to the <area> ΔΖ. Let the <area> ΓΘ be added <as> common; therefore the whole ΑΘ is equal to the gnomon ΜΝΞ. But the <area> ΑΘ is the <rectangle contained> by the <lines> ΑΔ, ΔΒ; for the <line> ΔΘ is equal to the <line> ΔΒ; therefore the gnomon ΜΝΞ, too, is equal to the <rectangle contained> by the <lines> ΑΔ, ΔΒ. Let the <area> ΛΗ be added <as> common (which is equal to the <square> on the <line> ΓΔ); therefore the gnomon ΜΝΞ and the <area> ΛΗ are equal to the rectangle contained by the <lines> ΑΔ, ΔΒ and the square on the <line> ΓΔ; but the gnomon ΜΝΞ and the <area> ΛΗ, <as a> whole, is the square ΓΕΖΒ, which is <the square> on the <line> ΓΒ; therefore the rectangle contained by the <lines> ΑΔ, ΔΒ, with the square on the <line> ΓΔ, is equal to the square on the <line> ΓΒ.

Therefore if a straight line is cut into equal and unequal <segments>, the rectangle contained by the unequal segments of the whole, with the square on the <line> between the cuts, is equal to the square on the half; which it was required to prove.

As you can see from Example 2a, Babylonian mathematics was treated in a similar way. More recently historians have been concerned to recover *how* the ancients thought about mathematics. In order to recover ancient mathematical *concepts*, historians have gone back to the original sources and retranslated them in a way which tries to stay as faithful as possible to the texture of the original vocabulary and syntax. To use translators' jargon, the mid-twentieth century translations are *domesticating*, in that they aim to make ancient mathematics familiar and comfortable. The newer translations, on the other hand, are *alienating*, in that they try to maintain the intellectual distance between the sources and us. As Reviel Netz, another of David's friends and protégés, has recently summarized:

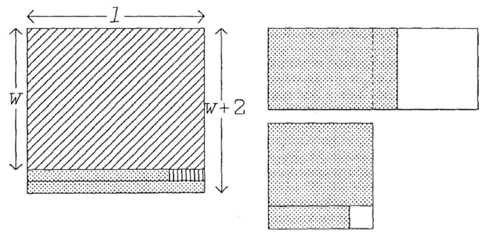
> There are many possible barriers to the reading of a text in a foreign language, and the purpose of a scholarly translation is as I understand it to remove all barriers having to do with the foreign language itself, leaving all other barriers intact. (Netz 2004: 3)

Example 2b: *AO 8862, Larsa, southern Iraq, c.1750* BCE *(trans. and comm. Høyrup 2002: 164–5, slightly amended)*

(1) Length, width. I have made length and width hold each other. I have built a surface. I turned around (it). As much as length went beyond width, I have appended to inside the surface: 3 03. I turned back. I have accumulated length and width: 27. What are the length, width, and surface?

27 3 03 the things accumulated
15 the length
12 the width 3 00 the surface

You, by your proceeding, append 27, the things accumulated, length and width, to inside 3 03: 3 30. Append 2 to 2: 29. You break its moiety, that of 29: 14;30 steps of 14;30 is 3 30;15. From inside 3 30;15 you tear out 3 30: 0;15, the remainder. The equal-side of 0;15 is 0;30. You append 0;30 to one 14;30: 15, the length. You tear out 0;30 from the second 14;30: 12, the true width. I have made 15, the length, and 12, the width, hold each other: 15 steps of 12 is 3 00, the surface. By what does 15, the length, go beyond 12, the width? It goes beyond by 3. Append 3 to inside 3 00, the surface: 3 03, the surface.



The text starts by stating that a rectangular surface or field (obliquely hatched in the diagram [on the left]) is built, that is, marked out: after pacing off its dimensions, the speaker 'appends' the excess of the length over the width (regarded as a broad line — vertically hatched) to it; the outcome is 3 03. Even this is done quite concretely in the terrain. then he 'turns back' and reports to accumulation of the length and the width to be 27. the procedure starts by 'appending' these latter 'things accumulated' (dotted) to the hatched surface, from which we get a new rectangle with the same length and width that has been augmented by 2. The surface is 3 03 + 27 = 3 30, and the sum of the length and the new width is obviously 27 + 2 = 29. This standard problem is solved by means of the usual procedure, as shown in the [diagram on the right]. The length turns out to be 15, and the width 14; the original or 'true' width is therefore 12.

In this, you might be feeling, Høyrup and Netz succeed all too well: there is no doubt that their translations are harder to read than their predecessors'. But although we have to work at understanding these new translations, the rewards are satisfying and often surprising. For instance, here is Tybjerg on Netz's findings (see Example 1b):

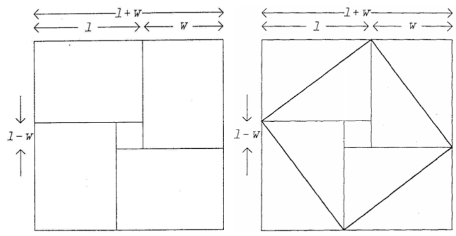
> The diagram and the proof are shown to form an interdependent unit, where neither can be understood without the other. Greek mathematics does not deal with abstract mathematical

entities, but rather with real geometrical objects in the real geometrical space of the diagram. Contrary to modern practice, the letters of the diagram are not symbols for general features of geometrical objects (as *c* is the centre of a circle in modern notation), but provide direct links between text and diagram. (Tybjerg 2005)

Example 3b: *BM 34568, Babylon, southern Iraq, c.300 BCE (trans. and comm. Høyrup 2002: 395–7), translation slightly amended*

(9) I have accumulated the length and the width: 14, and 48 the surface. Since you do not know, 14 steps of 14, 3 16. 48 steps of 4, 3 12. You lift 3 12 from 3 16: 4 is remaining. What steps of what may I go so that 4 (results)? 2 steps of 2, 4. You lift 2 from 14: 12 is remaining. 12 steps of 0;30, 6. The width is 6. You join 2 to 6: 8. The length is 8.

(15) The length goes beyond the width by 7 cubits. The surface is 2 00. What are the length and width? 2 00 steps of 4, 8 00. 7 steps of 7, 49. You join 8 00 and 49 with each other: 8 49. What steps of what may I go so that 8 49 (results)? 23 steps of 23, 8 49. You lift 7 from 23: 16 is remaining. You go 16 steps of 0;30, 8. The width is 8. Join 7 and 8: 15. The length is 15.



#9 and #15 are not new as problems, but their method is innovative. The procedure is based on the principle shown in [the left-hand figure above]. In #9 the complete square $\square(l+w)$ is found to be 3 16, from which 4 times the area [](l,w) is removed, that is, 3 12; this leaves $\square(l-w) = 4$, whence $l - w = 2$. Subtracting this difference from $l + w = 14$ leaves twice the width. The width is therefore 6, and addition of the difference gives the length. Not the faintest trace of the method of average and deviation is left, nor of any cut-and-paste procedure.

#15 is strictly analogous […], using the same configuration but with inserted diagonals [see right-hand figure above], as demonstrated by the apparently roundabout calculation of $23^2 - 2\bullet(23^2 - 17^2)$ instead of $2\bullet 17^2 - 23^2$. At first the square on the diagonal $\square(d) = 4\ 49$ is lifted from $\square(l+w) = 8\ 49$, which leaves 4 00 s the area of 4 half-rectangles. This remainder is doubled, which tells us that 4 times the rectangular area is 8 00 — and lifting this again from 8 49 leaves 49 for $\square(l-w)$. Thereby we are brought to the situation of #9, and the rest follows exactly the same pattern.

Høyrup, on the other hand, has shown that Old Babylonian mathematics, underneath its arithmetical surface is geometrical in a different way (see Example 2b). All the apparently arithmetical numbers have dimension: they are particular lengths and areas that are manipulated very physically. Note, for instance, that there are two forms of addition. Similar objects can be 'accumulated'—the literal meaning of the verb is 'to heap up'—and small objects can be 'appended' to larger ones. In this case, a line is added to an area by implicitly giving it a width of 1; in other contexts that conversion is explicitly made. Similarly, objects can be torn apart, or made to hold each other to form an area. And although there are no such cut-and-paste diagrams attached to Old

Babylonian mathematical problems, it is clear that they were implicit in the text. David had a lovely aphorism that 'Greek mathematics is to draw a figure and tell a story about it' (in Taisbak 1999). Høyrup shows us that Old Babylonian maths is at one level to *not* draw a figure but tell a story about it anyway.

In the Later Babylonian mathematics however, as Høyrup notes in the commentary to Example 3b, cut-and-paste procedures have disappeared completely from the repertoire of available techniques. Rather, the areas of two figures are compared: nothing is accumulated, nothing torn out or held together. The mathematical objects remain geometrical figures with specific lengths and areas, but mathematically we simply observe and compare them, whereas their Old Babylonian predecessors were actively interfered with. But even their geometrical status is unstable: Old Babylonian mathematics distinguishes quite clearly between making four identical copies of a single object (here the rectangle) and multiplying lengths and widths to make areas. Here, both operations are treated simply as arithmetical multiplication with the phrase 'steps of'.

In sum, then, we are dealing with two, perhaps three, very different mathematical cultures, even if we dismiss matters of language, script, media, and numeration as surface presentation. Old Babylonian mathematics is inherently metric: all parameters have both quantity and measure, explicit or implicit, as well as dimension. Later Babylonian mathematics is increasingly arithmetic, as geometrical operations are replaced by arithmetical ones Both varieties, however, are entirely inductive: solutions to specific problems serve as generic examples from which generalisations are inferred (not always correctly); and starting assumptions (axioms or postulates) are not stated explicitly. In contrast, the classical Greek tradition is inherently geometric: parameters have dimension but no quantity or measure. It is also heavily deductive and axiomatic: the emphasis is on deriving general proofs from explicitly stated theorems and axioms.

So on internal evidence alone we can dismiss Old Babylonian mathematics as the putative arithmetical precursor to early Greek mathematics, simply because it was not particularly arithmetical. Later Babylonian mathematics, on the other hand, seems to be our prime suspect at the moment: it fits the mathematical desiderata and, quite crucially, the chronological one too. Although our particular example probably just post-dates Euclid, the mathematical culture itself goes back several centuries earlier. We need now to step outside the mathematics itself to look at its context, and this is where the ignorance and indifference come in.

Ignorance and indifference: the social context

We have seen that Heath, Neugebauer, and Van Der Waerden were interested in discovering *what* mathematics was known in the ancient past. More recently Reviel Netz and Jens Høyrup have been concern to recover *how* mathematics was thought about and discussed. For me, though, and I think for David, it is just as important to consider *who* was doing mathematics and *why*. I have borrowed this formulation of historiography in terms of questions, by the way, from the historian of Chinese mathematics Qu Anjing (2002).

Let's start, as before with Greek mathematicians, as studied by David, and more recently by Reviel Netz and Serafina Cuomo, another of David's protégés. Mainland Greece supported a literate culture from perhaps as early as the eighth century BC, but literate mathematics as an intellectual discipline developed only in period 440–360, in and around Athens. According to Cuomo, our direct evidence for this early mathematics is patchy and circumstantial: material evidence of professional numeracy from accountants, land surveyors, and architects; the use of mathematical imagery and ideas by historians like Herodotus, playwrights like Aristophanes, and philosophers like Plato and Aristotle (Cuomo 2001: 4–38; Netz 1999: 272–277). Indeed Plato's dialogue *Meno*, in which Socrates leads an uneducated slave boy to think carefully about ratio and proportion, formed a cornerstone of David's argument in *The mathematics of Plato's academy* (1987/1999: 3–29).

The earliest Greek mathematician whose work has come down to us directly is Autolycus of Pitane, on the Turkish coast of the Aegean, who probably lived between about 360 and 290 BC and whose extant work is a geometrical treatise on astronomy (Cuomo 2001: 79). Earlier work, such as that by Hippocrates of Chios (c. 425 BCE?), is known only through descriptions by Simplicius and Eutocius, commentators of the sixth century CE. Their sources may have included a history of mathematics by Eudemus (c. 325 BCE), now lost (Fowler 1987/1999: 7). During Autolycus' lifetime, between 336 and 323 BCE, Alexander the Great's conquests overturned the political configuration of the Mediterranean and Middle East. After Alexander's death, his vast empire was divided into three separate kingdoms, centred on Egypt, Babylonia and Asia Minor. The following century was the golden age of Greek mathematics. Archimedes was active in Sicily, Euclid and Apollonius in Egypt, at the library and museum founded for Alexander himself.

Yet if we consider the eastern Mediterranean as a whole, we do not find society at large caught up in the excitement of the new, deductive-axiomatic mathematics. Reviel Netz, after a careful and imaginative survey of ancient Greek mathematicians as a social group, concludes:

> The quadrivium is a myth. Very few bothered at all in antiquity with mathematics, let alone became creative mathematicians. (Netz 1999: 289)

A detailed prosopographical study leads him to suggest three mathematicians—good, bad, and indifferent—were born at a rate of at most three a year across the antique Mediterranean world (Netz 1999: 285; see also Netz 2002). They tended to be from leisured, wealthy backgrounds with private incomes that allowed them to pursue their very unfashionable interests.

> [Mathematics] was an enterprise pursued by *ad hoc* networks of amateurish autodidacts — networks for which the written form is essential; constantly emerging and disappearing, hardly ever obtaining any institutional foothold. … Our expectations of a 'scientific discipline' should be forgotten. An 'intellectual game' will be a closer approximation. (Netz 1999: 291–292)

Turning now to Old Babylonian mathematics and its practitioners, we see a completely different social picture. Numeracy and mathematics had strong pre- and proto-historic roots in southern Iraq, where it was deeply embedded in quantitative techniques developed for the institutional management of the world's first urban states (Nissen *et al.* 1993). Towards the end of the third millennium BC, the development of the powerful and flexible sexagesimal place value system, initially just a streamlined calculation tool for overworked bureaucrats, opened up the possibility of a suprautilitarian mathematics. We have already seen an example of cut-and-paste algebra, but the corpus equally comprises problems two and three-dimensional geometry, and pseudo-practical scenarios (see Robson 1999: chs. 3–7).

The vast majority of our evidence comes from the eighteenth century BCE, around the time of king Hammurabi of Babylon. I have been particularly concerned to study mathematical tablets from well-documented archaeological excavations, in order to elucidate their social and intellectual functions. I have had particular success with an unprepossessing building, called 'House F' by its American excavators in the 1950s, from the city centre of ancient Nippur, near modern Kerbala (Robson 2001; 2002). During the 1740s BCE the house was used a scribal school, and at some point DIY repairs were made using the tablets from the house itself. Piecing together the excavated finds — often literally — now scattered across three museum collections in Iraq and America, I discovered that about 10 percent of the tablets used in the school were mathematical. I was able to place the learning of arithmetical tables, weights and measures, and mathematical problem-solving within the school's curriculum, and to show that each type of tablet had a particular pedagogical function.

Further, a lot of mathematical knowledge was instilled through the medium of Sumerian literature, along with clear messages about the ideology of just and effective government through careful use of mathematical methods. The message was often dressed up attractively, however. In a composition often known as 'The dialogue between Girini-isag and Enki-manshum' although it is more of a rumbustious slanging match, the advanced student Girini-isag belittles and humiliates his younger colleague Enki-manshum (whose defences are often rather ineffectual):

> (*Girini-isag speaks*): "You wrote a tablet, but you cannot grasp its meaning. You wrote a letter, but that is the limit for you! Go to divide a plot, and you are not able to divide the plot; go to apportion a field, and you cannot even hold the tape and rod properly; the field pegs you are unable to place; you cannot figure out its shape, so that when wronged men have a quarrel you are not able to bring peace but you allow brother to attack brother. Among the scribes you (alone) are unfit for the clay. What are you fit for? Can anybody tell us?"
>
> (*Enki-manshum replies*): "Why should I be good for nothing? When I go to divide a plot, I can divide it; when I go to apportion a field, I can apportion the pieces, so that when wronged me have a quarrel I soothe their hearts and […]. Brother will be at peace with brother, their hearts […]." (Following lines lost). (Vanstiphout 1997: 589)

Girini-isag's point is that accurate land surveys are needed for legal reasons — inheritance, sales, harvest contracts, for instance. If the surveyor cannot provide his services effectively he will unwittingly cause disputes or prevent them from being settled peacefully.

There is no archaeological or textual evidence of mathematics as a leisure pursuit or prestige occupation in the early second millennium BCE: it was taught to some scribal students, but not all of them, as part of their professional formation.

The collapse of the Old Babylonian state in 1600 BCE entailed a massive rupture of all sorts of scribal culture. Much of Sumerian literature was lost from the stream of tradition, it seems, and most of Old Babylonian mathematics too. When, in the later second millennium BC, cuneiform culture began to spread west to Egypt and the eastern Mediterranean coast, mathematics did not travel with it. At least, there is no evidence at all of any mathematical activity in any of the cuneiform scribal schools of Syria and the Levant, and precious little metrology (e.g., van Soldt 1995).

So Old Babylonian mathematics cannot have influenced early Greek developments: it was part of a scribal culture that all but died out nearly a millennium before the earliest Greek literate culture, 1200 miles away. The much-neglected Later Babylonian mathematics, however, is circumstantially a much better candidate to be seen as proto-Greek: our evidence dates from the mid-seventh century BC onwards, comfortably predating the earliest Greek sources and stories, and belongs to a world in which Greece is much closer to Mesopotamia than it was in the early second millennium BC. This time I will argue that it is highly likely that Greeks came into contact with Babylonian mathematics, but that they were generally indifferent to it (with the late exception of mathematical methods in astronomy).

We have already seen that there is precious little we can say about Greek mathematics before about 440 BCE, and that it probably matured into the recognisably 'Greek' style over the following century. Since the late sixth century mainland Greece had been continually threatened by the enormous Persian empire, which encompassed all of the Middle East including Egypt, Turkey, and of course Babylonia. Not surprisingly, we find that Greek writers have very little good to say about Persia and Babylonia at this period. They were fascinated and repelled in equal measure, and most accounts came in garbled form from travellers to the Persian court, or with the Persian army. Amongst the propaganda about effete eastern potentates and barbaric social practices there were stories of astronomers and magicians, but the Greek intelligentsia found very little in these narratives worth engaging with (Kuhrt 1995).

The situation changed to some extent after the conquests of Alexander the Great. Much of the Middle East was settled by Greek colonies, and ethnically Greek rulers and elites spread Greek culture and social practices, especially in the cities. But it remained a thin Greek veneer over a deeply rooted indigenous civilisation. In Babylon, for instance, Alexander's successors built a Greek theatre and hippodrome, but the heart of the city remained the temple of the god Marduk, who had been worshipped there for over one and a half thousand years. This temple, and the temple of the sky god Anu in the southern city of Uruk, continued to be bastions of Babylonian religion, literacy, and civilisation for generations after the rest of the populace had become more or less Persianised and later Hellenised (van der Spek 1985; 1987; 2001).

All known Babylonian mathematical activity, in the Persian and Hellenistic periods, is associated with these two temples. Whereas in the Old Babylonian period we saw mathematics as a training for a bureaucratic scribal class, by this time most institutional administration was carried out in Aramaic or Greek, only indirect evidence for which survives in southern Iraq. The temples, however, employed a class of priests called *kalû*, or lamenters, whose function was to ritually weep and wail and bang drums during solar and lunar eclipses in order to propitiate the gods and drive away any evil portended by the eclipse (Linssen 2004). But for that reason it was necessary to predict extremely accurately the timings of all ominous celestial events. Throughout the Persian and Hellenistic periods a sophisticated mathematical astronomy grew up which modelled the timings of eclipses and conjunctions. It was based on a mass of observational data, collected in the temples over centuries, but also on the calculational power of the sexagesimal system. The *kalû* priests thus needed serious mathematical training. Indeed, where it is possible to identify the owners and authors of Later Babylonian mathematical tablets, even those like our example which have nothing to do with astronomy, the signatures on the tablets and/or the archaeological context invariably indicate that they were written and read by *kalû* priests, or their close colleagues the *ashipu* priests (Rochberg 1993).

Now, as many of you know, a great deal of Babylonian mathematical astronomy, and presumably mathematics too, reached the Greek astronomical community—or at least the Greek astronomer Hipparchus—in the second century BCE, probably via Alexandria (e.g., Jones 1991; 1993; 1996). But that was a century too late for Euclid and his contemporaries. Earlier than that the Greeks remained largely indifferent to Babylonian culture, convinced of their own intellectual supremacy. Most famously, a man named Bel-Re'ushunu, known in Greek as Berossos, the administrator of Marduk's temple during the 250s BCE, wrote a history of Babylonia in Greek, for a Greek audience. It seems to have sunk like a lead balloon; at least, none of the later historians who cite it are ethnically Greek. Fragments of it survive in Armenian and Jewish writings, and that is all (Verbrugghe and Wickersham 1996).

Inferences

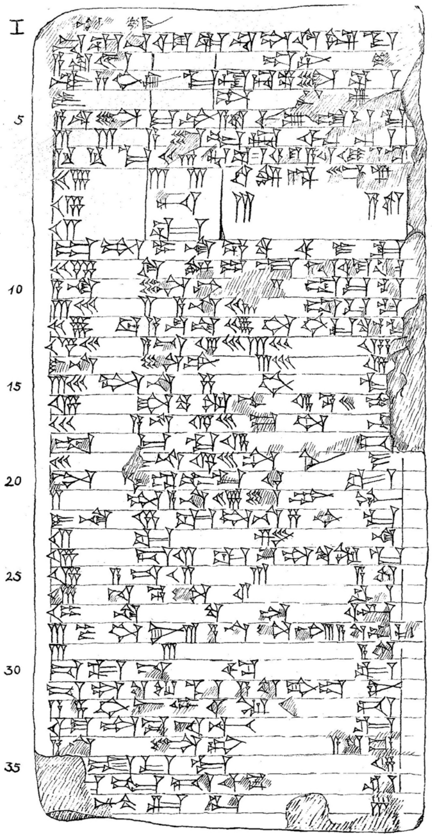
We have been examining three mathematical traditions maintained and transmitted across centuries, by tiny communities of experts. Ancient historians often use the phrase 'the stream of tradition' to describe the continuity of Babylonian intellectual culture. But as we have seen, in the mathematical case at least, those streams are more like trickles, liable to dry up at any moment. It is hardly surprising that the mathematicians of Classical antiquity should have been entirely ignorant of the achievements of Old Babylonian mathematics (as indeed were their Later Babylonian successors), over a thousand miles away and a millennium before. Equally, the dwindling, isolated, conservative community of numerate astronomer-priests in Hellenistic Babylonia had little in common with the patrician leisured mathematicians of the contemporary Mediterranean.

The modern community of historians of ancient mathematics is similarly tiny — indeed it makes Reviel Netz's estimate of three Greek mathematicians a year seem enormous. But fortunately for us, twenty-first century society, institutions, and communications mean that we will be neither ignorant of nor indifferent to David and his work. He will continue to influence us in many profound ways for decades to come.

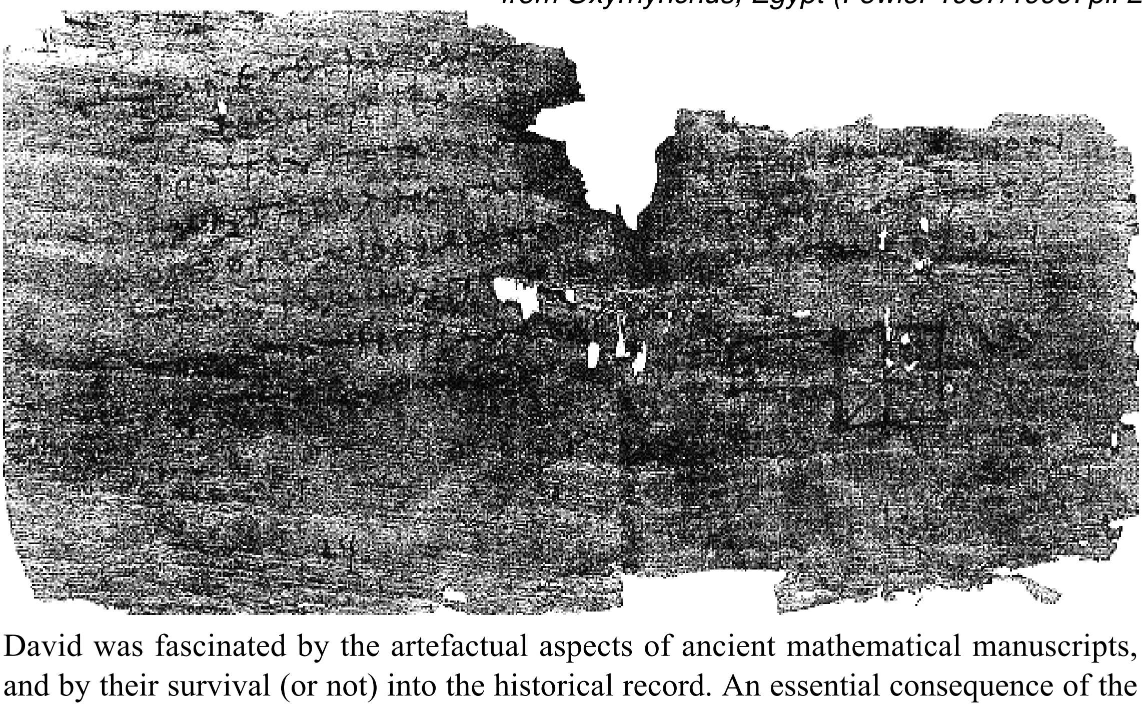
Acknowledgements

This is a slightly amended version of the talk given at the memorial symposium for David Fowler, Mathematics Institute, University of Warwick, 9 November 2004 (see Steve Russ's report on pages 41–43), and the British Society for the History of Mathematics Christmas Meeting, Birmingham, 11 December 2004 (see Benjamin Wardhaugh's report on pages 37–38). I warmly thank Miles Reid, Jeremy Gray, and Elaine Greaves Coelho for their help in organising the first meeting, and June Barrow-Green for reading this on my behalf at the second. But most of all I thank Denise.

Appendix: on the materiality of textual objects



Above left: AO 8862, column I (Neugebauer 1935–37: II pl. 35); above right: BM 34568 (Neugebauer 1935–37: III pl. 1); below: POxy i.29, a fragmentary ancient manuscript of Elements *II.5 from Oxyrhynchus, Egypt (Fowler 1987/1999: pl. 2)*



David was fascinated by the artefactual aspects of ancient mathematical manuscripts, and by their survival (or not) into the historical record. An essential consequence of the fact that for most historians of ancient mathematics, most of the time, our primary sources are not the ancient documents themselves but nineteeth- or twentieth-century translations and modernisations, is that the embodied experience of reading them — handwritten on a tablet or a scroll or a potsherd, with no punctuation, mediating paratext, or word division — can never be replicated by reading from a printed book.

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